CM02 FORCES IN CIRCULAR MOTION

SPH4U

EQUATIONS

- Force with centripetal acceleration

$$\Sigma F = \frac{mv^2}{r}$$

FORCE FOR CENTRIPETAL ACCELERATION

- Recall:

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 r f^2$$

- Since Newton's 2nd Law states $\Sigma F = ma$, we get the following formulas for the force's magnitude

$$\Sigma F = \frac{mv^2}{r} = \frac{4\pi^2 mr}{T^2} = 4\pi^2 mrf^2$$

EXAMPLE 1

A car of mass 1.1×10^3 kg negotiates a level curve at a constant speed of 22 m/s. The curve has a radius of 85 m, as shown in **Figure 2**.

(a) Draw an FBD of the car and name the force that provides the centripetal acceleration.

(b) Determine the magnitude of the force named in (a) that must be exerted to keep the car from skidding sideways.

(c) Determine the minimum coefficient of static friction needed to keep the car on the road.

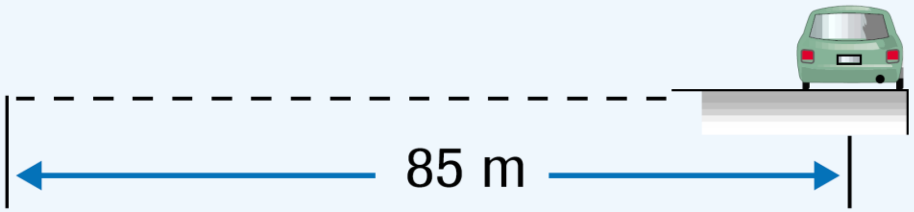


Figure 2
The radius of the curve is 85 m.

EXAMPLE 1 – SOLUTIONS

(a) **Figure 3** is the required FBD. The only horizontal force keeping the car going toward the centre of the arc is the force of static friction ($\vec{F}_S$) of the road on the wheels perpendicular to the car's instantaneous velocity. (Notice that the forces parallel to the car's instantaneous velocity are not shown in the FBD. These forces act in a plane perpendicular to the page; they are equal in magnitude, but opposite in direction because the car is moving at a constant speed.)

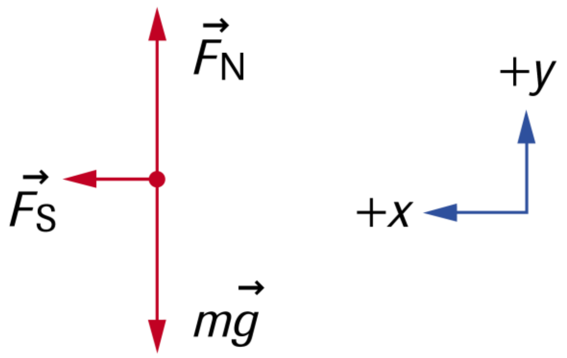


Figure 3
The FBD of the car on a level curve

EXAMPLE 1 – SOLUTIONS CONT.

(b) $m = 1.1 \times 10^3$ kg

$v = 22$ m/s

$r = 85$ m

$F_S = ?$

$$F_S = \frac{mv^2}{r}$$

$$= \frac{(1.1 \times 10^3 \text{ kg})(22 \text{ m/s})^2}{85 \text{ m}}$$

$$F_S = 6.3 \times 10^3 \text{ N}$$

The magnitude of the static friction force is 6.3×10^3 N.

EXAMPLE 1 – SOLUTIONS CONT.

(c) $g = 9.8$ N/kg

We know from part (b) that the static friction is 6.3×10^3 N and from **Figure 3** that $F_N = mg$. To determine the minimum coefficient of static friction, we use the ratio of the maximum value of static friction to the normal force:

$$\mu_S = \frac{F_{S,max}}{F_N}$$

$$= \frac{6.3 \times 10^3 \text{ N}}{(1.1 \times 10^3 \text{ kg})(9.8 \text{ N/kg})}$$

$$\mu_S = 0.58$$

The minimum coefficient of static friction needed is 0.58. This value is easily achieved on paved and concrete highways in dry or rainy weather. However, snow and ice make the coefficient of static friction less than 0.58, allowing a car travelling at 22 m/s to slide off the road.

EXAMPLE 2

A car of mass 1.1×10^3 kg travels around a frictionless, banked curve of radius 85 m. The banking is at an angle of 19° to the horizontal, as shown in **Figure 4**.

(a) What force provides the centripetal acceleration?

(b) What constant speed must the car maintain to travel safely around the curve?

(c) How does the required speed for a more massive vehicle, such as a truck, compare with the speed required for this car?

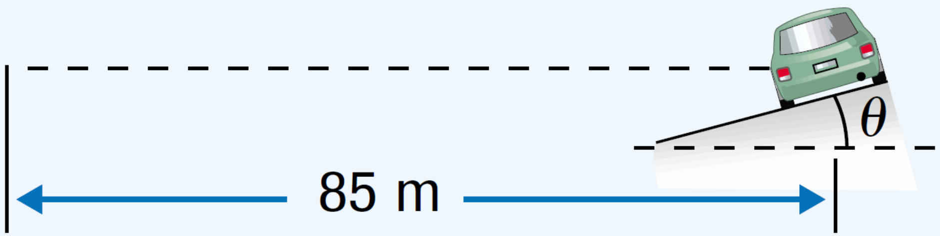


Figure 4
The radius of the curve is 85 m.

EXAMPLE 2 – SOLUTIONS

(a) From the FBD shown in **Figure 5**, you can see that the cause of the centripetal acceleration, which acts toward the centre of the circle, is the horizontal component of the normal force, $F_N \sin \theta$. Thus, the horizontal acceleration, a_x, is equivalent to the centripetal acceleration, a_c. (Notice that the FBD resembles the FBD of a skier going downhill, but the analysis is quite different.)

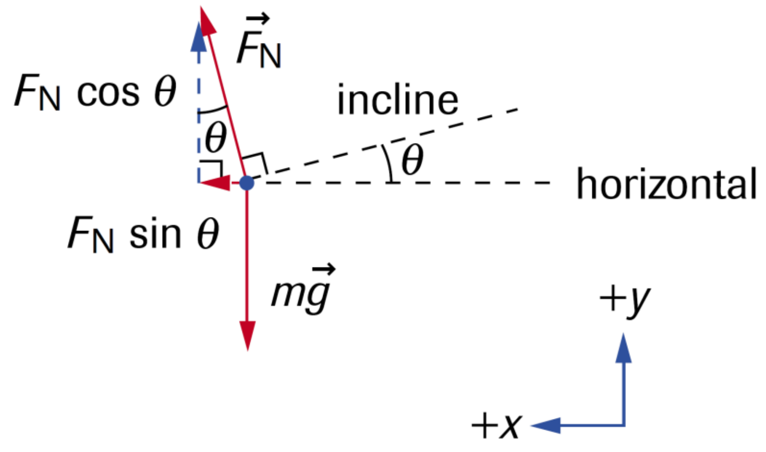


Figure 5
The FBD of the car on a banked curve

EXAMPLE 2 – SOLUTIONS CONT.

(b) $m = 1.1 \times 10^3$ kg

$r = 85$ m

$\theta = 19°$

$v = ?$

We take the vertical components of the forces:

$$\sum F_y = 0$$

$$F_N \cos\theta - mg = 0$$

$$F_N = \frac{mg}{\cos\theta}$$

EXAMPLE 2 – SOLUTIONS CONT.

Next, we take the horizontal components of the forces:

$$\sum F_x = ma_c$$

$$F_N \sin\theta = ma_c$$

$$\frac{mg}{\cos\theta}\sin\theta = ma_c$$

$$mg\tan\theta = \frac{mv^2}{r}$$

$$v^2 = gr\tan\theta$$

$$v = \pm\sqrt{gr\tan\theta}$$

$$= \pm\sqrt{(9.8\ \text{m/s}^2)(85\ \text{m})(\tan 19^\circ)}$$

$$v = \pm 17\ \text{m/s}$$

We choose the positive square root, since v cannot be negative. The speed needed to travel safely around the frictionless curve is 17 m/s. If the car travels faster than 17 m/s, it will slide up the banking; if it travels slower than 17 m/s, it will slide downward.

EXAMPLE 2 – SOLUTIONS CONT.

(c) The speed required for a more massive vehicle is the same (17 m/s) because the mass does not affect the calculations. One way of proving this is to point to our expression $v^2 = gr \tan\theta$: v depends on g, r, and θ but is independent of m.

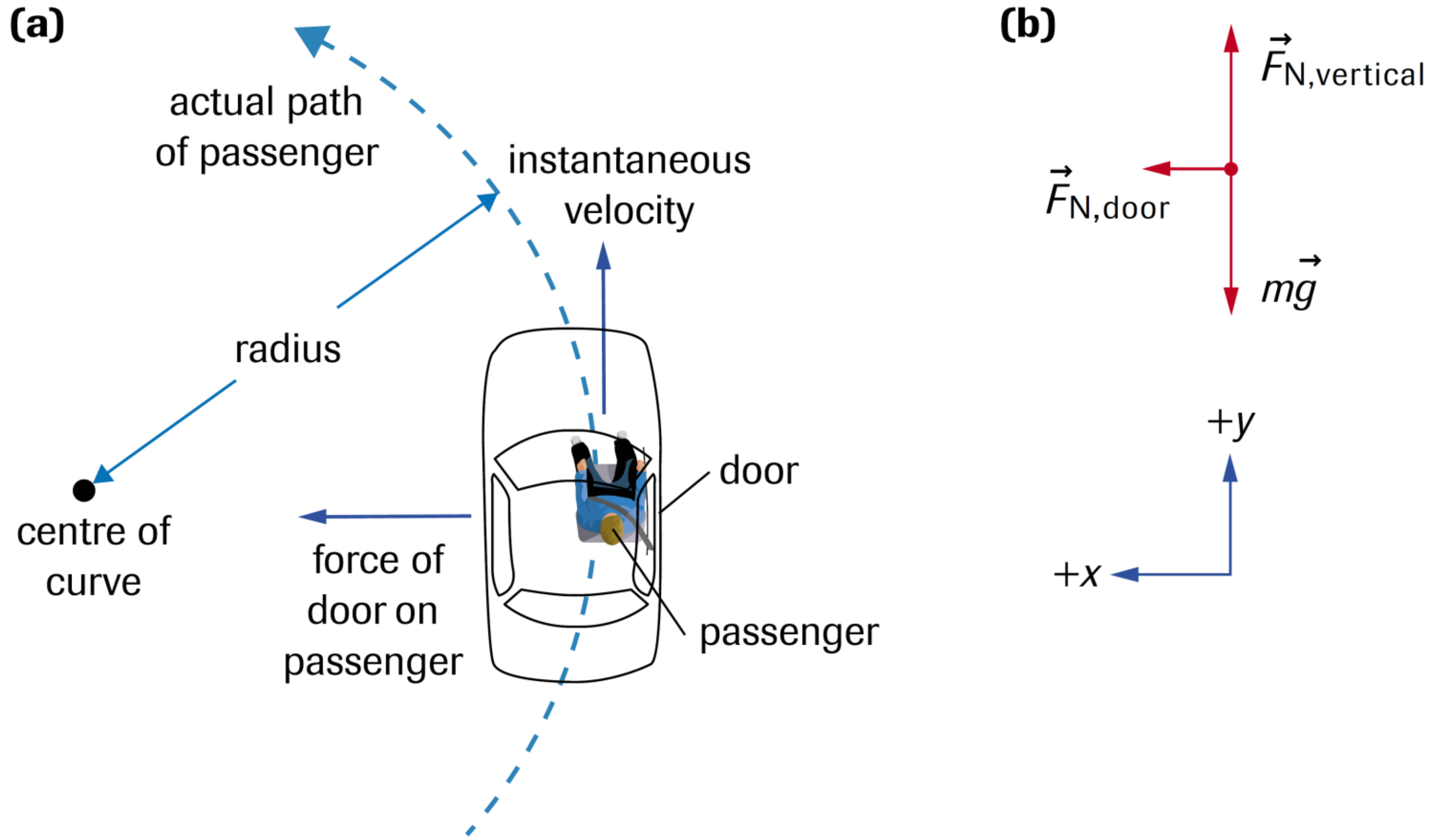
ROTATING FRAMES OF REFERENCE

- When using the Earth as an inertial reference frame:

Figure 10

(a) The top view of a passenger in a car from Earth's frame of reference as the car makes a left turn

(b) The side-view FBD of the passenger



ROTATING FRAMES OF REFERENCE

- Recall: The Law of Inertia (Newton's 1st Law) does not hold in an accelerating (noninertial) reference frame

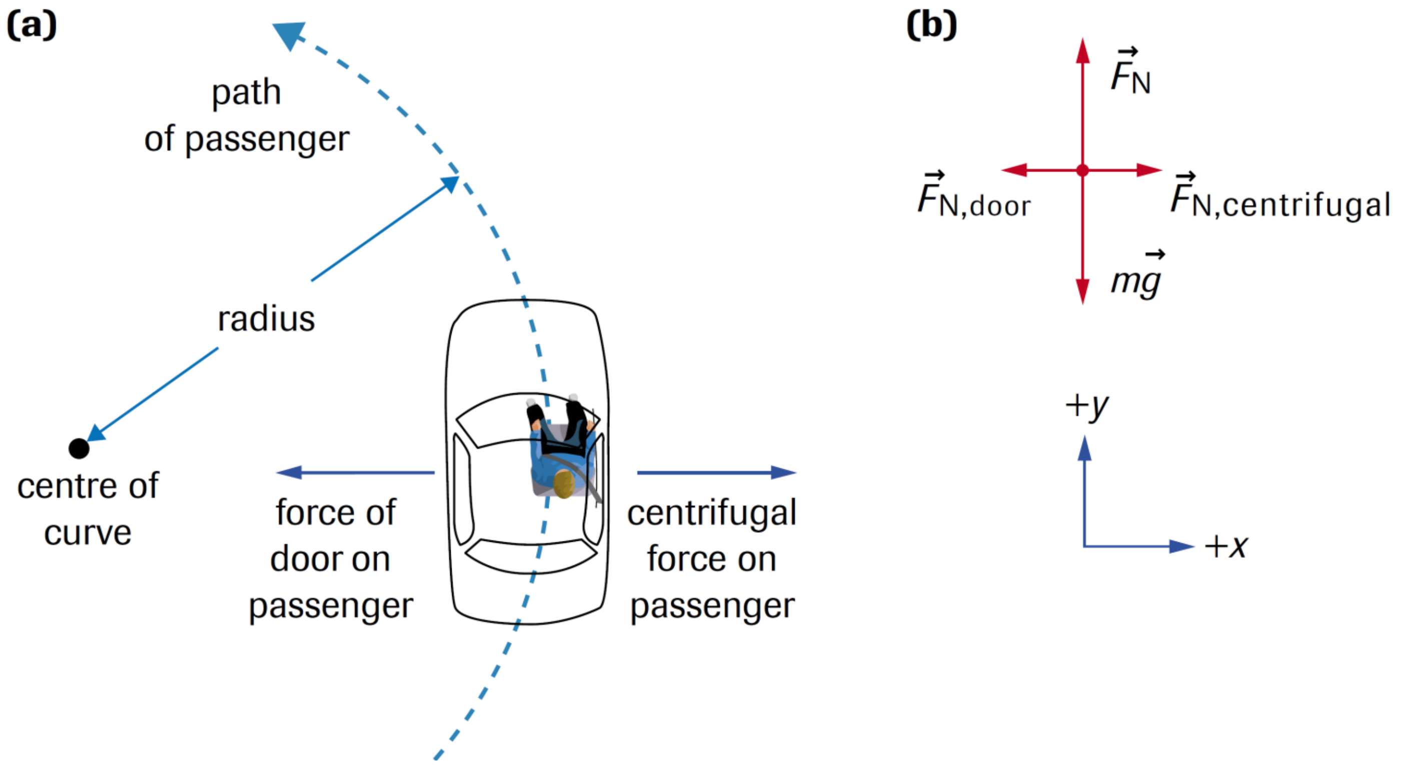


Figure 11

(a) Top view of a passenger from the car's frame of reference as the car makes a left turn

(b) The side-view FBD of the passenger, showing the fictitious force in the accelerating frame of reference

ROTATING FRAMES OF REFERENCE

- **Centrifugal Force:** force in a rotating (accelerating) frame of reference
- Application: **Centrifuge** – rapidly-rotating device used for separating substances and training astronauts

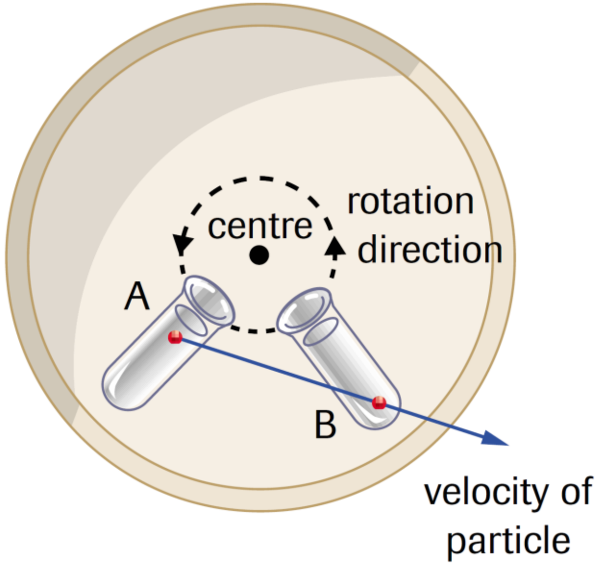


Figure 13
As the centrifuge rotates, a particle at position A tends to continue moving at a constant velocity, thus settling to the bottom of the tube.

ROTATION FRAMES OF REFERENCE

- **Coriolis Force:** fictitious force that acts perpendicular to the velocity of an object in a rotating frame of reference
- This force has effects on objects moving very quickly or for very long time periods:
  - Weather patterns are greatly affected
  - Firing long-range projectiles – need to recalibrate sights to account for the Coriolis Force of the area

SUMMARY

- The net force acting on an object in uniform circular motion acts toward the centre of the circle. (This force is sometimes called the centripetal force, although it is always just gravity, the normal force, or another force that you know already.)
- The magnitude of the net force can be calculated by combining Newton's second law equation with the equations for centripetal acceleration.
- The frame of reference of an object moving in a circle is a noninertial frame of reference.
- Centrifugal force is a fictitious force used to explain the forces observed in a rotating frame of reference.
- Centrifuges apply the principles of Newton's first law of motion and centrifugal force.
- The Coriolis force is a fictitious force used to explain particles moving in a rotating frame of reference.

PRACTICE

Readings

- Section 3.2 (pg 128)

Questions

- pg 138 #1-9